PHYC/ECE 463 Advanced Optics I

Fall 2007

Homework #9, Due Wednesday Nov. 7

1-Problem 5.7 (K&F)

2-With regard to Young's double slit experiment, derive a general expression for the shift in the vertical position of the *m* the maximum as a result of placing a thin sheet of glass with index *n* and thickness *d* directly over one of the slits. Identify your assumptions.

3-Problem 5.22 (K&F)

4- White light falling on two long narrow slits emerges and is observed in a distant screen. If red light (λ_0=*780 nm*) in the first order fringe overlaps violet in the second order fringe, what is the latter's wavelength?

H.W # 9 Solutions

#1 K&F 5.7

$$\langle S \rangle = \langle S_1 \rangle + \langle S_2 \rangle + 2\sqrt{\langle S_1 \rangle \langle S_2 \rangle}\, \cos\delta$$

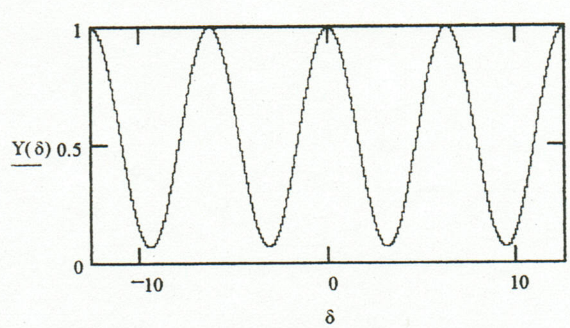
or

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\cos\delta \qquad \text{where } \delta \simeq \frac{2\pi a}{\lambda}\frac{x'}{D'}$$

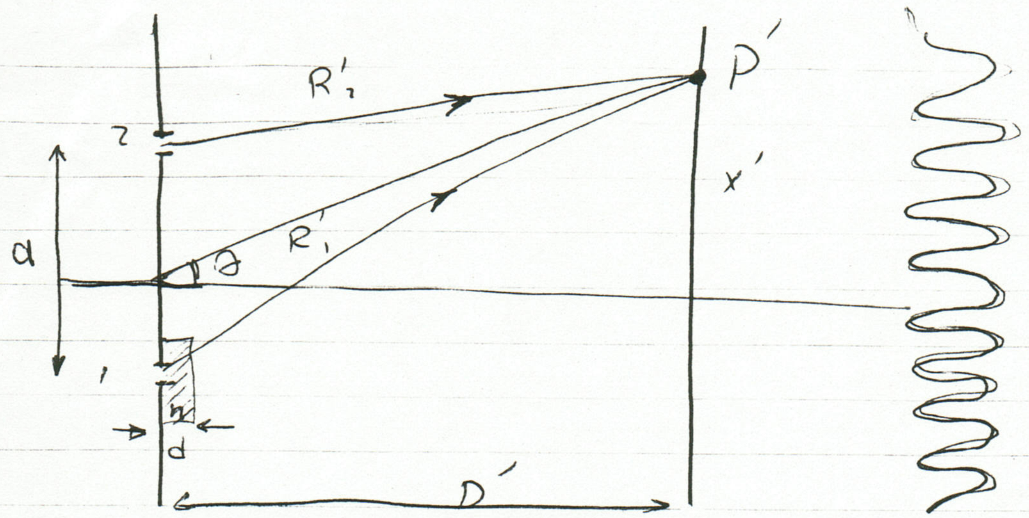
Let $I_2 = \frac{1}{3} I_1$ (due to attenuation filter)

$$I = 3I_2 + I_2 + 2\sqrt{3}\, I_2 \cos\delta = 4I_2 \left(1 + \frac{\sqrt{3}}{2}\cos\delta\right)$$

$$I = 2(2+\sqrt{3})\, I_2 \times \underbrace{\frac{\left(1 + \frac{\sqrt{3}}{2}\cos\delta\right)}{1 + \frac{\sqrt{3}}{2}}}_{Y(\delta)}$$



#2



Assumptions: ignore multiple reflection in the glass, small θ

at P'

$$\Phi_2 = \frac{2\pi}{\lambda} R'_2$$

$$\Phi_1 \approx \frac{2\pi}{\lambda}(R'_1 - d) + \frac{2\pi}{\lambda} d\, n$$

$$\delta = \Phi_1 - \Phi_2 = \frac{2\pi}{\lambda}(R'_1 - R'_2) + \frac{2\pi}{\lambda} d(n-1)$$

$$\delta = \frac{2\pi}{\lambda}\left[a\frac{x'}{D'} + d(n-1)\right]$$

Since bright stripes correspond to $\delta = 2m\pi$

$$\frac{a x'}{D'} + d(n-1) = m\lambda \quad \Rightarrow \quad x'_m = m\lambda\frac{D'}{a} + d\frac{D'}{a}(n-1)$$

Thus, all the fringes (regardless of order m) are shifted upward by

$$\boxed{\Delta x' = d\frac{D'}{a}(n-1)}$$

#3 K&F 5.22

$d = 2\,mm \qquad n = 1.6 \qquad \lambda_0 = 500\,nm$

$$m_{max} = \frac{2nd}{\lambda_0} + \frac{1}{2} = \frac{2\times1.6\times0.2\,cm}{500\times10^{-7}cm} + \frac{1}{2} \simeq 12800.5$$

$$m = m_{max}\left(1 - \frac{1}{2}\theta_2^2\right) \simeq m_{max}\left(1 - \frac{1}{2}\frac{\theta_1^2}{n^2}\right)$$

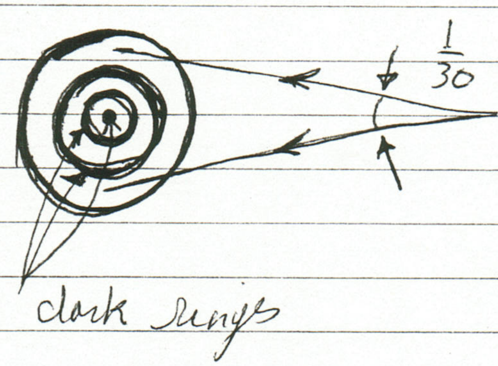
$$P(\#\text{ of fringes}) = m_{max} - m = m_{max}\frac{(\Delta\theta_1)^2}{2n^2}$$

for $\Delta\theta_2 = \frac{1}{30}$

$$P = 12800 \times \frac{1}{900\times2(1.6)^2} \approx 2.77$$

2 full fringes?

3 dark rings (including the central spot).



Note

Since m_{max} = half integer
The central spot is dark

#4 ✓

The mth order fringe associated with $\underline{\lambda}$ is given by

$$\frac{\pi}{\lambda} a \frac{x'}{D'} = m\pi \qquad \text{or} \qquad x'_m = m\lambda \frac{D'}{a}$$

$$x'_1(\lambda = 780) = x'_2(\lambda = ?)$$

$$780 \frac{D'}{a} = 2\lambda \frac{D'}{a} \qquad \lambda = \frac{780}{2} = 390 \text{ nm}$$